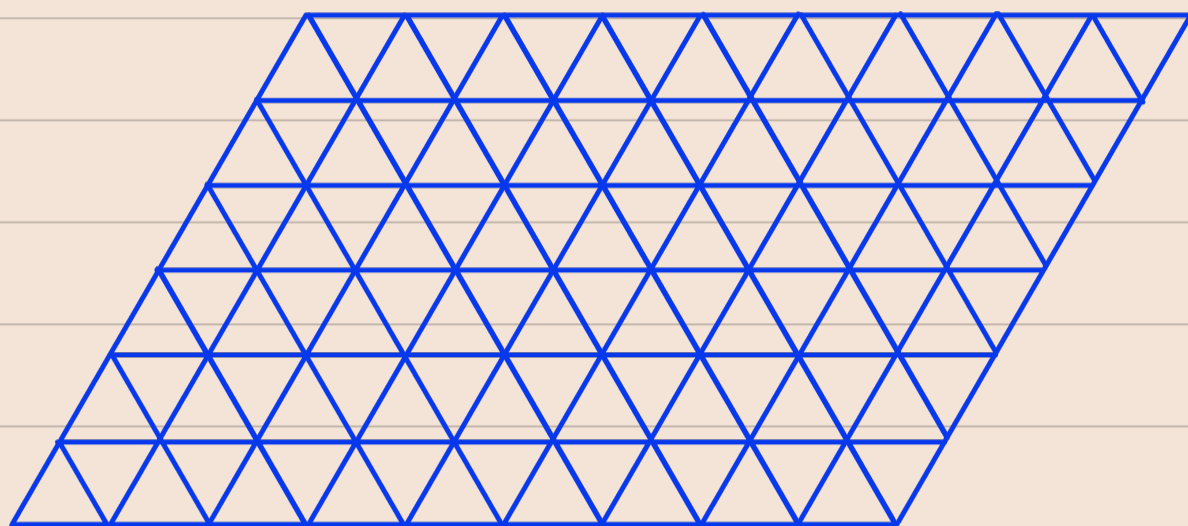


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Lecture 26

RG for 2D Ising Ferromagnet on a triangular lattice



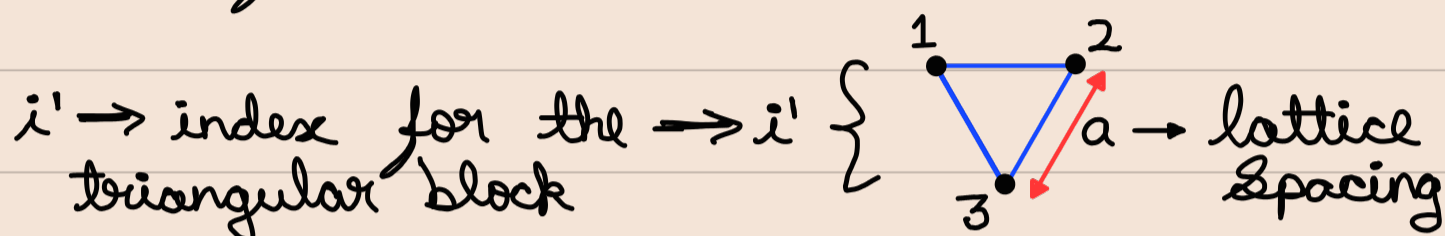
Consider Ising spins arranged in a 2D triangular lattice with only nearby spin ferromagnetic interaction.

In the above lattice, the spins are situated in the vertices of the triangles. The hamiltonian of the system is given by,

$$\tilde{\mathcal{H}} = -J \sum_{\langle ij \rangle} S_i S_j - h \sum_i S_i \quad \& \quad \mathcal{H} = -\beta \tilde{\mathcal{H}} \quad \text{dimensionless hamiltonian}$$

$$\Rightarrow \mathcal{H} = K \sum_{\langle ij \rangle} S_i S_j + L \sum_i S_i$$

The appropriate coarse graining procedure here is the Kadanoff's block spin approach where we can consider a block of three spins in a triangular cell of the lattice.



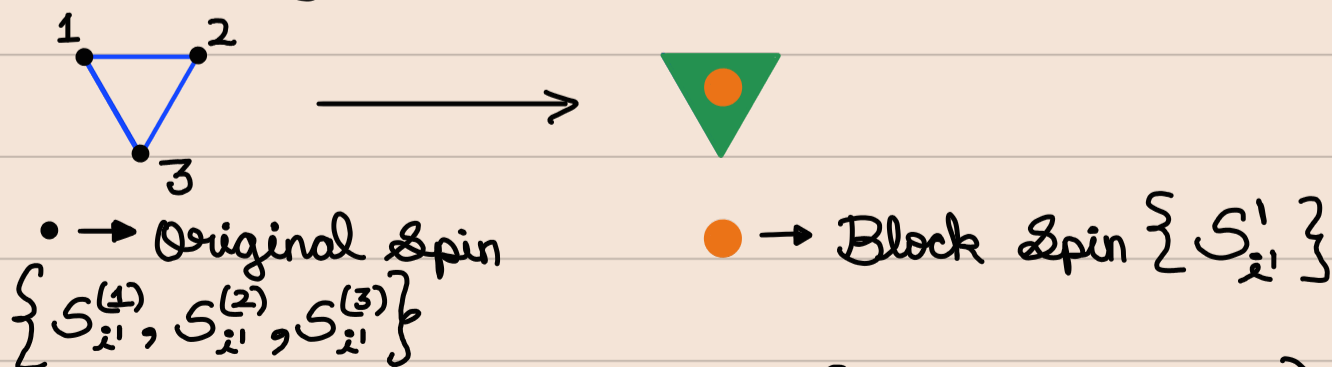
Each block consists of three spins.

$S_{i'}^{(1)} \rightarrow$ top left of i'^{th} triangle

$S_{i'}^{(2)} \rightarrow$ top right of i'^{th} triangle

$S_{i'}^{(3)} \rightarrow$ bottom of i'^{th} triangle

The block spin can be assigned by the majority rule.



Majority rule: $\underline{S_{i'}'} = \text{sgn}(S_{i'}^{(1)} + S_{i'}^{(2)} + S_{i'}^{(3)})$

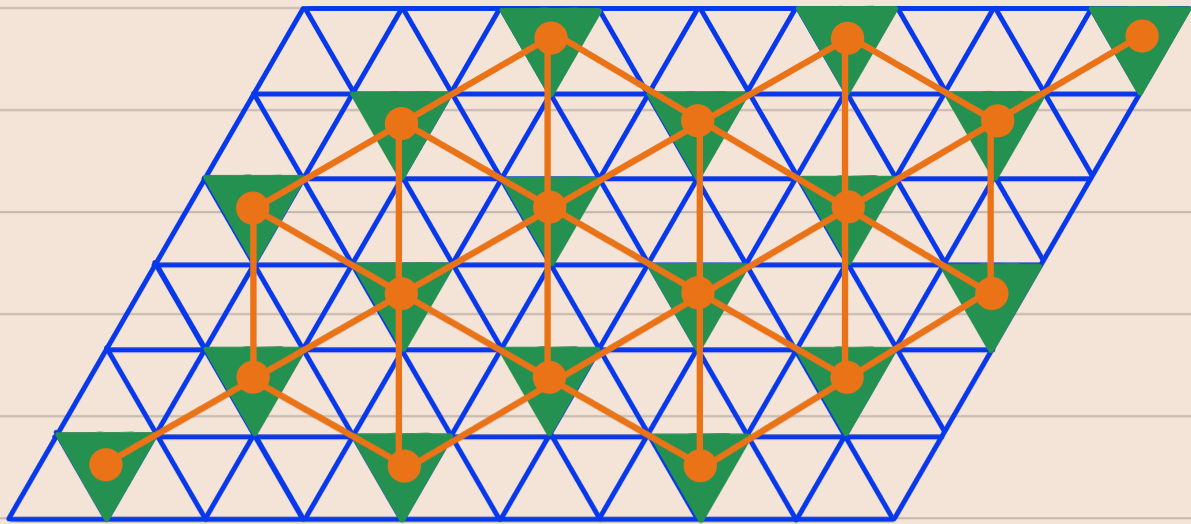
We define the majority assigning indicator operator as,

$$\hat{T}[\{S_{i'}'\}; \{S_{i'}^{(4)}, S_{i'}^{(2)}, S_{i'}^{(3)}\}]$$

$$= \prod_{i'} \frac{1}{2} \left[1 + \frac{S_{i'}' (S_{i'}^{(4)} + S_{i'}^{(2)} + S_{i'}^{(3)} - S_{i'}^{(4)} S_{i'}^{(2)} S_{i'}^{(3)})}{2} \right]$$

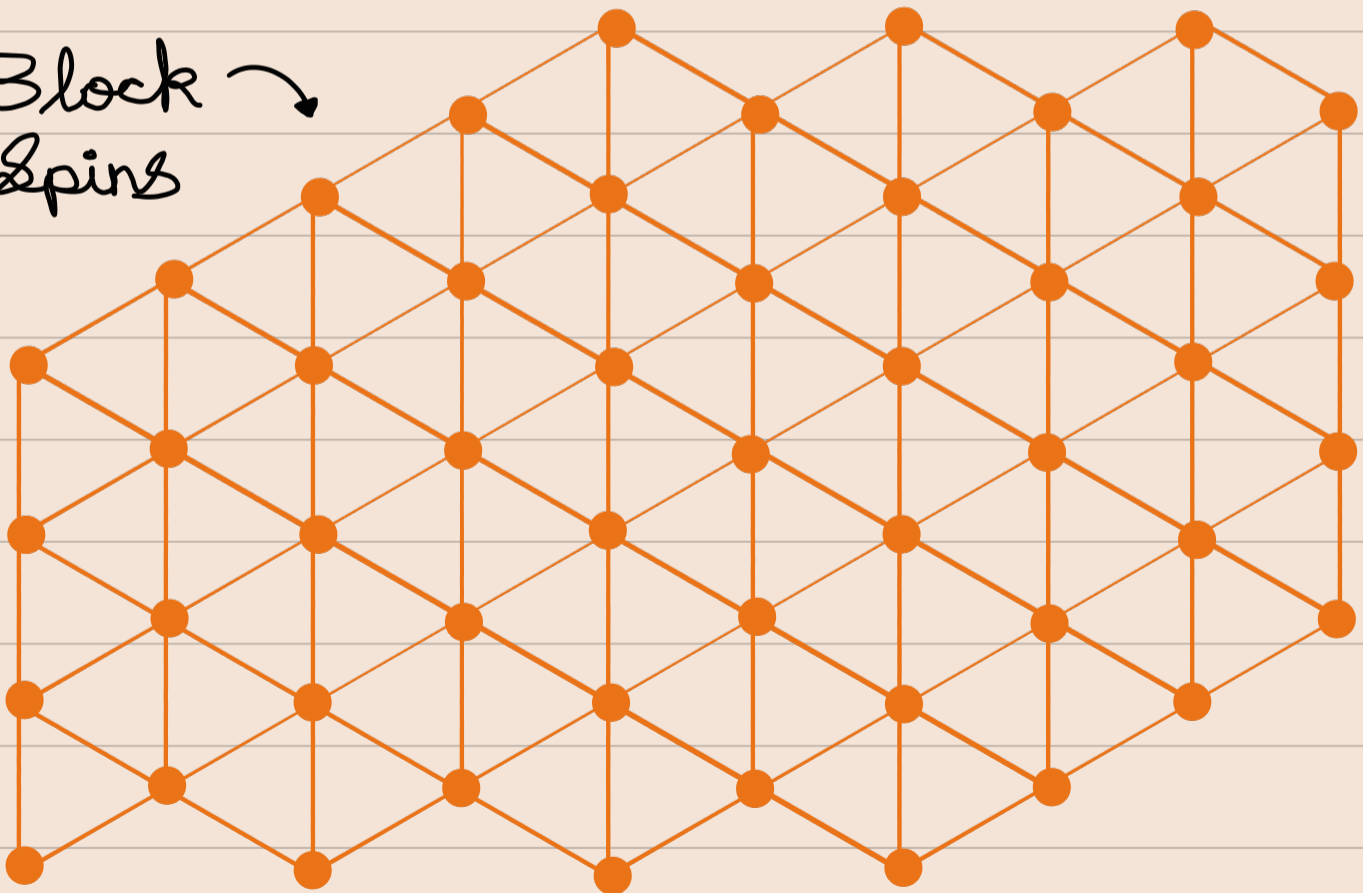
$S_{i'}^{(4)}$	$S_{i'}^{(2)}$	$S_{i'}^{(3)}$	$S_{i'}'$	$\hat{T}$
1	1	1	1	1
1	1	-1	1	1
1	-1	-1	1	0
-1	-1	-1	1	0
1	1	1	-1	0
1	1	-1	-1	0
1	-1	-1	-1	1
-1	-1	-1	-1	1

Thus $\hat{T}[\{S_{i'}'\}; \{S_{i'}^{(4)}, S_{i'}^{(2)}, S_{i'}^{(3)}\}]$ is '1' if the majority rule is satisfied in all blocks & Zero if even one block doesn't obey majority rule.



Blue line correspond to original spins in triangular lattice & orange (bigger) dots are block spins which also results in another triangular lattice with a slight global rotation & a bigger size compared to original.

Block
Spins →



The block spins are rotated by 30° compared to original spin lattice & the new lattice constant size is given by $a' = \sqrt{3} a \rightarrow b = \sqrt{3}$.

Coarse graining can be mathematically shown by the transformation

$$e^{H'\{\mathcal{S}'\}} = \text{Trace}_{\{\mathcal{S}\}} \left[\hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{H\{\mathcal{S}\}} \right]$$

This maintains $Z'(H') \propto Z(H)$.

We expect the entire interaction to be split into two parts where

$$H = H_0 + V$$

where H_0 is the interactions inside the block & V for the interactions between the blocks.

The Zeroth order hamiltonian which describes the interactions within a cell, neglecting external interaction is given by,

$$\mathcal{H}_0\{\mathcal{S}'\} = K \sum_{i'} (S_{i'}^{(1)} S_{i'}^{(2)} + S_{i'}^{(2)} S_{i'}^{(3)} + S_{i'}^{(3)} S_{i'}^{(1)}) + L \sum_{i'} (S_{i'}^{(1)} + S_{i'}^{(2)} + S_{i'}^{(3)})$$

The partition function of these blocks without any external is,

$$\mathcal{Z}'_0 = \sum_{\{\mathcal{S}'\}} e^{\mathcal{H}_0\{\mathcal{S}'\}}$$

$$= \sum_{\{\mathcal{S}'\}} e^{\left(\sum_{i'} \left[K(S_{i'}^{(1)} S_{i'}^{(2)} + S_{i'}^{(2)} S_{i'}^{(3)} + S_{i'}^{(3)} S_{i'}^{(1)}) + L(S_{i'}^{(1)} + S_{i'}^{(2)} + S_{i'}^{(3)}) \right] \right)}$$

$$= \sum_{\{\mathcal{S}'\}} \prod_{i'} e^{\left[K(S_{i'}^{(1)} S_{i'}^{(2)} + S_{i'}^{(2)} S_{i'}^{(3)} + S_{i'}^{(3)} S_{i'}^{(1)}) + L(S_{i'}^{(1)} + S_{i'}^{(2)} + S_{i'}^{(3)}) \right]}$$

$$= \prod_{i'} \sum_{S_{i'} = \{\pm 1\}} e^{\left[K(S_{i'}^{(1)} S_{i'}^{(2)} + S_{i'}^{(2)} S_{i'}^{(3)} + S_{i'}^{(3)} S_{i'}^{(1)}) + L(S_{i'}^{(1)} + S_{i'}^{(2)} + S_{i'}^{(3)}) \right]}$$

$$= \prod_{i'} \mathcal{Z}_{i'}$$

$\mathcal{Z}_{i'}$ = Partition function of i'^{th} block

If N' is the total number of blocks in the lattice & if $Z'_{i'} = Z'_0$ is same for all blocks, then

$$Z'_0 = (Z'_0)^{N'}, \text{ where,}$$

$$Z'_0 = \sum_{S'_{i'} = \{\pm 1\}} e^{\left[K(S_{i'}^{(1)} S_{i'}^{(2)} + S_{i'}^{(2)} S_{i'}^{(3)} + S_{i'}^{(3)} S_{i'}^{(1)}) + L(S_{i'}^{(1)} + S_{i'}^{(2)} + S_{i'}^{(3)}) \right]}$$

Let $\mathcal{H}_{i'}$ be the hamiltonian of i^{th} block,
 $\rightarrow \mathcal{H}_{i'}(S'_{i'}) = K(S_{i'}^{(1)} S_{i'}^{(2)} + S_{i'}^{(2)} S_{i'}^{(3)} + S_{i'}^{(3)} S_{i'}^{(1)}) + L(S_{i'}^{(1)} + S_{i'}^{(2)} + S_{i'}^{(3)})$

There are 8 possible original spins which gives rise to two block spins.

$S_{i'}^{(1)}$	$S_{i'}^{(2)}$	$S_{i'}^{(3)}$	$S'_{i'}$	$e^{\mathcal{H}_{i'}\{S'\}}$
1	1	1	1	e^{3K+3L}
1	1	-1	1	e^{-K+L}
1	-1	1	1	e^{-K+L}
-1	1	1	1	e^{-K+L}
1	-1	-1	-1	e^{-K-L}
-1	1	-1	-1	e^{-K-L}
-1	-1	1	-1	e^{-K-L}
-1	-1	-1	-1	e^{3K-3L}

$$S'_{i_1} = 1 \text{ when } \text{sgn}(S_{i_1}^{(1)} + S_{i_1}^{(2)} + S_{i_1}^{(3)}) = 1$$

$$\& S'_{i_1} = -1 \text{ when } \text{sgn}(S_{i_1}^{(1)} + S_{i_1}^{(2)} + S_{i_1}^{(3)}) = -1$$

$$Z'_0 = e^{3K+3L} + 3e^{-K+L} + 3e^{-K-L} + e^{3K-3L}$$

$$\rightarrow Z'_0 = 2 \left[e^{3K} \cosh(3L) + 3e^{-K} \cosh(L) \right]$$

We also define Z'_+ & Z'_- as

$$Z'_+ = \sum_{\substack{\{S_1, S_2, S_3\} \\ S' = \text{sgn}(S_1 + S_2 + S_3) = 1}} e^{\mathcal{H}'_{i_1} \{S'\}} = e^{3K+3L} + 3e^{-K+L}$$

$$\& Z'_- = \sum_{\substack{\{S_1, S_2, S_3\} \\ S' = \text{sgn}(S_1 + S_2 + S_3) = -1}} e^{\mathcal{H}'_{i_1} \{S'\}} = e^{3K-3L} + 3e^{-K-L}$$

$$\Rightarrow Z'_0 = Z'_+ + Z'_-$$

The hamiltonian $\mathcal{H}'_0$ can also be written as a hamiltonian for a set of non non-interacting block spins as,

$$\mathcal{H}'_0 = \sum_{i'} (h'_0 S'_i + g'_0)$$

$\hookrightarrow \text{Constant}$

$$\Rightarrow Z'_0 = \sum_{\{S'_i\}} e^{\mathcal{H}'_0 \{S'_i\}} = \prod_{i'} \sum_{S'_{i'} = \pm 1} e^{h'_0 S'_{i'} + g'_0}$$

$$\text{But } Z'_0 = \prod_{i'} (Z'_0) = \prod_{i'} (Z'_+ + Z'_-)$$

$$\begin{aligned} \Rightarrow Z'_+ + Z'_- &= \sum_{S'_{i'} = \pm 1} e^{h'_0 S'_{i'} + g'_0} \\ &= e^{h'_0 + g'_0} + e^{-h'_0 + g'_0} \end{aligned}$$

$$\text{Also } Z'_+ = \underbrace{e^{h'_0 + g'_0}}_{S'_{i'} = 1} \quad \& \quad Z'_- = \underbrace{e^{-h'_0 + g'_0}}_{S'_{i'} = -1}$$

$$\Rightarrow Z'_+ Z'_- = e^{2g'_0} \Rightarrow \underline{g'_0 = \frac{1}{2} \ln(Z'_+ Z'_-)}$$

$$\& \underline{\frac{Z'_+}{Z'_-} = e^{2h'_0} \Rightarrow h'_0 = \frac{1}{2} \ln\left(\frac{Z'_+}{Z'_-}\right)}$$

Using $Z'_\pm$ we can also find the average values of spins inside a block a function of block spin $S'_{i'}$. We know $\langle S^{(1)}_{i'} \rangle_{(S'_{i'} = \pm 1)} = \frac{\sum S^{(1)}_{i'} e^{\mathcal{H}'_{i'_{\pm}}}}{Z'_\pm}$

If $S_{i'}^1 = 1$, the possible configurations of $\{S_{i'}^{(1)}, S_{i'}^{(2)}, S_{i'}^{(3)}\}$ are as follows,

$S_{i'}^{(1)}$	$S_{i'}^{(2)}$	$S_{i'}^{(3)}$	$S_{i'}^1$	$e^{\mathcal{H}_{i'}^1 \{S_{i'}^1\}}$
1	1	1	1	e^{3K+3L}
1	1	-1	1	e^{-K+L}
1	-1	1	1	e^{-K+L}
-1	1	1	1	e^{-K+L}

$$\Rightarrow \langle S_{i'}^{(1)} \rangle_0 (S_{i'}^1 = 1) = \frac{e^{3K+3L} + e^{-K+L}}{e^{3K+3L} + 3e^{-K+L}} = \frac{n_+}{z'_+}$$

The above result is same for all $S_{i'}^{(1)}, S_{i'}^{(2)}$ & $S_{i'}^{(3)}$. $\langle S_{i'}^{(1)} \rangle_0 (S_{i'}^1 = 1) = \frac{n_+}{z'_+}$
 where $n_+ = e^{3K+3L} + e^{-K+L}$

$$\& \langle S_{i'}^{(1)} \rangle_0 (S_{i'}^1 = -1) = - \frac{[e^{3K-3L} + e^{-K-L}]}{e^{3K-3L} + 3e^{-K-L}} = - \frac{n_-}{z'_-}$$

The above result is same for all $S_{i'}^{(1)}, S_{i'}^{(2)}$ & $S_{i'}^{(3)}$. $\langle S_{i'}^{(1)} \rangle_0 (S_{i'}^1 = -1) = - \frac{n_-}{z'_-}$
 where $n_- = e^{3K-3L} + e^{-K-L}$

We can write $\langle S_{i'}^{(1)} \rangle_0$ as an explicit function of $S_{i'}$ as

$$\langle S_{i'}^{(1)} \rangle_0 = \alpha S_{i'} + \beta$$

$$\Rightarrow \langle S_{i'}^{(1)} \rangle_0 (S_{i'} = 1) = \alpha + \beta = \frac{n_+}{z'_+}$$

$$\langle S_{i'}^{(1)} \rangle_0 (S_{i'} = -1) = -\alpha + \beta = \frac{-n_-}{z'_-}$$

$$\Rightarrow \alpha = \frac{1}{2} \left[\frac{n_+}{z'_+} + \frac{n_-}{z'_-} \right] \text{ \& } \beta = \frac{1}{2} \left[\frac{n_+}{z'_+} - \frac{n_-}{z'_-} \right]$$

$$\Rightarrow \alpha = \frac{1}{2} \left[\frac{e^{3K+3L} + e^{-K+L}}{e^{3K+3L} + 3e^{-K+L}} + \frac{e^{3K-3L} + e^{-K-L}}{e^{3K-3L} + 3e^{-K-L}} \right]$$

$$\text{ \& } \beta = \frac{1}{2} \left[\frac{e^{3K+3L} + e^{-K+L}}{e^{3K+3L} + 3e^{-K+L}} - \frac{e^{3K-3L} + e^{-K-L}}{e^{3K-3L} + 3e^{-K-L}} \right]$$

Thus $\langle S_{i'}^{(1)} \rangle_0 = \alpha S_{i'} + \beta$ is a function of $S_{i'}$ (block spin).

To find the total partition function when the system's original hamiltonian is separable into zero-th order & a small perturbation we use the Niemeijer-Van Leeuwen Cumulant approx.

$$H = H_0 + V$$

Intra-block interactions $\leftarrow$ $\rightarrow$ Inter-block interactions

$\Rightarrow$ The RG transform is

$$e^{H'\{\mathcal{S}'\}} = \text{Trace}_{\{\mathcal{S}\}} \left[\hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{H\{\mathcal{S}\}} \right]$$

$$= \text{Trace}_{\{\mathcal{S}\}} \left[\hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{H_0} \underbrace{(1 + V)}_{1^{\text{st}} \text{ order approx}} \right]$$

$$= \text{Trace}_{\{\mathcal{S}\}} \left[\hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{H_0} + V \hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{H_0} \right]$$

$$= \text{Tr}_{\{\mathcal{S}\}} \left[\hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{H_0} \right] + \text{Tr}_{\{\mathcal{S}\}} \left[V \hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{H_0} \right]$$

$$\begin{aligned} &\rightarrow e^{\mathcal{H}'\{\mathcal{S}'\}} \\ &= T_{\mathcal{H}_{\{\mathcal{S}\}}} \left[\hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{\mathcal{H}_0} \right] \left(1 + \frac{T_{\mathcal{H}_{\{\mathcal{S}\}}} [V \hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{\mathcal{H}_0}]}{T_{\mathcal{H}_{\{\mathcal{S}\}}} [\hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{\mathcal{H}_0}]} \right) \end{aligned}$$

$$\rightarrow e^{\mathcal{H}'\{\mathcal{S}'\}} = e^{\mathcal{H}_0'\{\mathcal{S}'\}} \left(1 + \frac{T_{\mathcal{H}_{\{\mathcal{S}\}}} [V \hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{\mathcal{H}_0}]}{T_{\mathcal{H}_{\{\mathcal{S}\}}} [\hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{\mathcal{H}_0}]} \right)$$

where $T_{\mathcal{H}_{\{\mathcal{S}\}}} [\hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{\mathcal{H}_0}] = e^{\mathcal{H}_0'\{\mathcal{S}'\}}$

$\mathcal{H}_0'\{\mathcal{S}'\}$ is the non-interacting part of Coarse Grained Hamiltonian.

$$\Rightarrow \mathcal{H}'\{\mathcal{S}'\} = \mathcal{H}_0'\{\mathcal{S}'\} + \log \left(1 + \frac{T_{\mathcal{H}_{\{\mathcal{S}\}}} [V \hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{\mathcal{H}_0}]}{T_{\mathcal{H}_{\{\mathcal{S}\}}} [\hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{\mathcal{H}_0}]} \right)$$

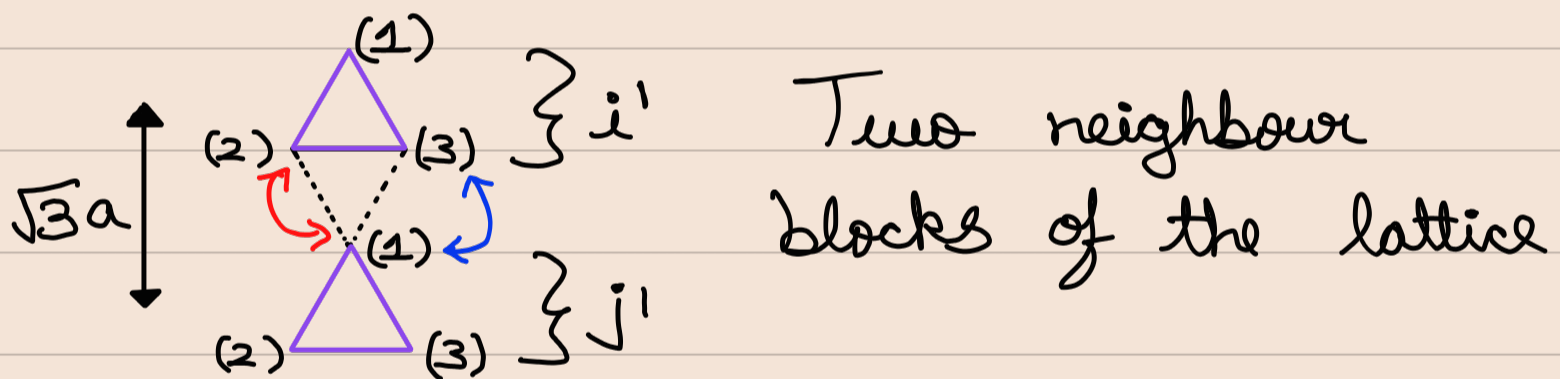
$$\left(\log(1+x) \sim x \quad V \text{ is small} \right)$$

$$\Rightarrow \mathcal{H}'\{\mathcal{S}'\} = \mathcal{H}_0'\{\mathcal{S}'\} + \frac{T_{\mathcal{H}_{\{\mathcal{S}\}}} [V \hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{\mathcal{H}_0}]}{T_{\mathcal{H}_{\{\mathcal{S}\}}} [\hat{T}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{\mathcal{H}_0}]}$$

Defining
$$\bar{V}\{\mathcal{S}'\} = \frac{\text{Tr}_{\{\mathcal{S}\}} \left[V \hat{\uparrow}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{\mathcal{H}_0} \right]}{\text{Tr}_{\{\mathcal{S}\}} \left[\hat{\uparrow}[\{\mathcal{S}'\}; \{\mathcal{S}\}] e^{\mathcal{H}_0} \right]}$$

$\bar{V}$ is the average of V taken with respect to non-interacting part of original hamiltonian given a config $\{\mathcal{S}'\}$.

$$\rightarrow \mathcal{H}'\{\mathcal{S}'\} = \mathcal{H}_0'\{\mathcal{S}'\} + \bar{V}\{\mathcal{S}'\}$$



$$\bar{V} = \sum_{\langle i', j' \rangle} \langle \text{Inter-triangle terms} \rangle$$

$$= \sum_{\langle i', j' \rangle} \left(K \langle S_{i'}^{(2)} \rangle_0 \langle S_{j'}^{(1)} \rangle_0 + K \langle S_{i'}^{(3)} \rangle_0 \langle S_{j'}^{(1)} \rangle_0 \right)$$

$$= K \sum_{\langle i', j' \rangle} 2 (\alpha S_{i'}' + \beta) (\alpha S_{j'}' + \beta)$$

$$\Rightarrow \bar{V} = 2K \sum_{\langle i', j' \rangle} \left[\alpha^2 S_{i'}' S_{j'}' + \alpha \beta (S_{i'}' + S_{j'}') + \beta^2 \right]$$

$$\Rightarrow \mathcal{H}' = \sum_{i'} (h'_0 S'_i + g'_0) + 2K \sum_{\langle i', j' \rangle} [\alpha^2 S'_{i'} S'_{j'} + \alpha \beta (S'_{i'} + S'_{j'}) + \beta^2]$$

$$\sum_{i', j'} S'_{i'} = \sum_{i'} \sum_{j' \in n \cdot n(i')} S'_{i'} = \sum_{i'} S'_{i'} \underbrace{\sum_{j' \in n \cdot n(i')} 1}_{\substack{\text{no. of nearest} \\ \text{neighbours}}} = 6 \sum_{i'} S'_{i'}$$

$$\Rightarrow \sum_{i', j'} S'_{i'} = \sum_{i', j'} S'_{j'} = 6 \sum_{i'} S'_{i'}$$

$$\mathcal{H}' = 2K\alpha^2 \sum_{i', j'} S'_{i'} S'_{j'} + (24K\alpha\beta + h'_0) \sum_{i'} S'_{i'} + (12K + g'_0) N'$$

The coarse grained Hamiltonian upto 1st order in V is,

$$\mathcal{H}' = K' \sum_{i', j'} S'_{i'} S'_{j'} + L' \sum_{i'} S'_{i'} + C' N'$$

where $K' = 2K\alpha^2$

$L' = 24K\alpha\beta + h'_0$

& $C' = 12K + g'_0$

$N' = \frac{N}{3}$ & $b = \sqrt{3}$

$$\alpha = \frac{1}{2} \left[\frac{e^{3K+3L} + e^{-K+L}}{e^{3K+3L} + 3e^{-K+L}} + \frac{e^{3K-3L} + e^{-K-L}}{e^{3K-3L} + 3e^{-K-L}} \right]$$

$$\beta = \frac{1}{2} \left[\frac{e^{3K+3L} + e^{-K+L}}{e^{3K+3L} + 3e^{-K+L}} - \frac{e^{3K-3L} + e^{-K-L}}{e^{3K-3L} + 3e^{-K-L}} \right]$$

$$h'_0 = \frac{1}{2} \ln \left(\frac{z_+}{z_-} \right) \quad \& \quad g'_0 = \frac{1}{2} \ln(z_+ z_-)$$

$$z'_+ = e^{3K+3L} + 3e^{-K+L}$$

$$z'_- = e^{3K-3L} + 3e^{-K-L}$$

$$z_0 = e^{3K+3L} + 3e^{-K+L} + 3e^{-K-L} + e^{3K-3L}$$

If $L=0$ ($h_{ext}=0$)

$$\rightarrow K' = 2K\alpha^2 \quad \& \quad \alpha = \left[\frac{e^{3K} + e^{-K}}{e^{3K} + 3e^{-K}} \right]$$

$$\beta = 0$$

$$z_+ = z_- = \frac{z_0}{2}, \quad h'_0 = 0, \quad g'_0 = \ln \left(\frac{z_0}{2} \right)$$

$$C' = 12K + g_0 = 12K + \ln(z_0) - \ln(2)$$

The recursion relation for k' is,
 $k' = 2k\alpha^2$ & for fixed point
 $k^* = 2k^*\alpha^{*2} \Rightarrow \alpha^{*2} = \frac{1}{2}$

$$\Rightarrow \frac{e^{3k^*} + e^{-k^*}}{e^{3k^*} + 3e^{-k^*}} = \frac{e^{4k^*} + 1}{e^{4k^*} + 3} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow e^{4k^*} + 1 = \frac{e^{4k^*}}{\sqrt{2}} + \frac{3}{\sqrt{2}}$$

$$\Rightarrow e^{4k^*} \left[1 - \frac{1}{\sqrt{2}} \right] = \frac{3}{\sqrt{2}} - 1$$

$$\Rightarrow e^{4k^*} = \frac{3 - \sqrt{2}}{\sqrt{2} - 1}$$

$$\Rightarrow k^* = \frac{1}{4} \log \left[\frac{3 - \sqrt{2}}{\sqrt{2} - 1} \right] \sim 0.33561$$

$$k' = 2k\alpha^2 \text{ \& } \alpha = \left[\frac{e^{3k} + e^{-k}}{e^{3k} + 3e^{-k}} \right] = \frac{e^{4k} + 1}{e^{4k} + 3}$$

$$k' = 2k \left[\frac{e^{4k} + 1}{e^{4k} + 3} \right]^2$$

$$\text{linearize} \Rightarrow \delta K' = \left(\frac{\partial K'}{\partial K} \right)_{K^*} \delta K$$

$$\frac{\partial K'}{\partial K} = 2 \left[\frac{e^{4K} + 1}{e^{4K} + 3} \right]^2 + \frac{32K e^{4K}}{(e^{4K} + 3)^2} \left[\frac{e^{4K} + 1}{e^{4K} + 3} \right]$$

$$= 2\alpha^2 + \frac{32K e^{4K}}{(e^{4K} + 3)^2} \alpha$$

$$\left. \frac{\partial K'}{\partial K} \right|_{K^*} = \frac{1 + 8 \log \left[\frac{3 - \sqrt{2}}{\sqrt{2} - 1} \right] \left(\frac{3 - \sqrt{2}}{\sqrt{2} - 1} \right) \frac{1}{\sqrt{2}}}{\left[\left(\frac{3 - \sqrt{2}}{\sqrt{2} - 1} \right) + 3 \right]^2}$$

$$\Rightarrow \left. \frac{\partial K'}{\partial K} \right|_{K^*} = 1.62352 = \Lambda_K^{(b)}$$

$$\Rightarrow \delta K^{(l)} = [1.62352]^l \delta K$$

$$\Rightarrow \delta K^{(l)} = b^{\lambda_T l} \delta K$$

$\lambda_T = \text{Thermal eigen value}$

$$\partial K^{(l)} = (\sqrt{3})^{\lambda_T l} \partial K$$

Since $\Lambda_K^{(b)} = b^{\lambda_T} \Rightarrow 1.62352 = (\sqrt{3})^{\lambda_T}$

$$\Rightarrow \lambda_T = \frac{\ln(1.62352)}{\ln(\sqrt{3})}$$

$$\Rightarrow \lambda_T \sim 0.88219$$

$$\& \nu = \frac{1}{\lambda_T} \Rightarrow \underline{\underline{\nu \sim 1.1335}}$$

The exact values of $K^* = 0.27465$ & $\nu = 1$ for 2D Ising model. Thus RG gave considerable estimates for K^* (or T^*) & ν .

The small deviation from $h_{\text{set}} = 0$ case can be done in the similar way.

In linear order of 'L' we get

$$\partial L' = 3 \left[\frac{e^{3K^*} + e^{-K^*}}{e^{3K^*} + 3e^{-K^*}} \right] \partial L = 3\alpha^*(\partial L)$$

$$ZL' = \frac{3}{\sqrt{2}}(ZL) \Rightarrow \Lambda_H^{(b)} = \frac{3}{\sqrt{2}}$$

$$\Rightarrow ZL' = b^{\lambda_H} ZL \quad (b = \sqrt{3})$$

$$\rightarrow \lambda_H = \frac{\log(3/\sqrt{2})}{\log(\sqrt{3})} \sim 1.36907$$

The exact value is 1.875.